9.HOMOMORPHISMs

(handout)



**Fig. 1**—Homomorphism understood on the backdrop of G as the original and H as the screen. The semiotic function that interests us here is how H is used to study/investigate G. In a homomorphism—trailing structural similarities across G and H—identity applies and maps across G and H, based on the premise that G and H are similar in some aspects, and different in others. However, if we consider G as the original and H as a screen, then image imh in H emerges truly as a map (icon) of and the kernel kerh as a print (index) in G.

*Homomorphism* is a mathematical concept—meaning *the same shape*—in algebra. It is a method for comparing groups on the basis of structural similarities. The two groups compared may be quite different which is why recurring shapes are of interest, not only to compare similar structures (as in mathematics) but also to investigate G through H. These may be similar in that the same shapes may be identified in both, but the comparison also contributes to make the difference between them specific, and investigative. In this handout we are interested in *transposing* homomorphism.

This option comes with the transposition to *semiotics*: for instance, we can define the groups G and H—in the above diagram—as *expression* and *content*, featuring in the investigative relationship between the *kernel* in G (*kerh*) and its image in H (*imh*). Notwithstanding its algebraic definition, and mathematical operations, it clearly *also* features a semiotic idea. To a kernel *kerh* of applicable structures in G is associated an image *imh* in H. We are interested in the image *imh* to the extent that it reveals a *content* of G in H. We are interested in *kerh* as the *expression* of H in G.

By focussing on this semiotic extension of homomorphism—the relation between *kerh* (expression) and *imh* (content)—we may not only take interest in stating the *similarity* between G and H (the premise that we depart from) but also make a point of their *difference*. The investigation of the properties of the groups G and H, in the light of the semiotic relation between the kernel (*kerh*) and image (*imh*) follows logically from the notion that *identities in G* map as *identities in H* (and



Fig. 2—Illuminated manuscript: Book of hours from the 15th century (Allegheny College). In its function of illumination the image needs not function as an illustration to work with the text in *print*. It invites a bi-modal reading in which the compound of text and image, in this case, is to give the hours a home. Not abstract hours but hours to inhabit. Given that this is an illuminated manuscript the print takes place in a reader who takes the acquired contents into possession, to whom this item marks the hours. In this sense, it makes the mark.

*inversions in G* map as *inversions in H*). In semiotic terms, homomorphism defines a relation between a map and a territory (or, alternatively, between <u>icon and index</u>).

Still in semiotic terms, the image *imh* is the group G in 'code'; while the kernel *kerh* is the group G 'in formation': here, the semiotics of code and the semiotics of signproduction are different, but there is no reason why they should not be integrated in *one* theory. If our design is not only to compare G and H, but to *study* G by the intermedium of H, then we can understand H as a *screen* and G as the *original*. The kernel *kerh* then is then defined as a structure in G that can be screened by H. We know that *identity* is preserved in G and H, they are also *similar* and so *different*.

We are now considering the image *imh* in its relation to *H* as a screen. While we are considering the kernel *kerh* in its relation to *G* as an original. At this point we are moving in the opposite direction from the homomorphism, in its mathematical reception in **Fig. 1**, to its possible semiotic materialisation in *print* (in the sense of mark-making, **Fig. 2**). Here, the homomorphism is *implied*, in the sense that we

## (handout)

may look for it, and not find it, in the relation between a text (as G) and an image (as H): instead we may will find it operating between the compound (as G) and the domestic framework of the reader (as H). This is a case in point of abduction: a form of inference defined by the fact that *not all* premises are known. In this sense, homomorphism *explains* abduction (a form of inference between deduction & induction; that is, from some premises and observation combined).

But abduction also *enriches* homomorphism in the sense that it leads us promptly to consider—and in some cases intercept—those aspects of the group G that featuring at the backdrop of the kernel *kerh*. It is in the dark of not only the structural corpus of G that departs from H, but stands effectively for itself (as *other*): aspects of G that are *incomparable* to H, that become *materially* specific in print. The sign in *production, code* and its *materialisation* in print, feature a kind of *distributed intelligence*, in which the *sign* extends what *language* is in thought: i.e. thought vs. extension.

Thought and extension *cannot* be conflated. But their vectorial sum can be intercepted intuitively (Spinoza): that is, it is specific and can be substantial. We are well acquainted with this: the evacuation of the *other*, amounts to the obliteration of substance. It causes a great trouble in present day academia: whether we are speaking scientific or artistic academia, we are making *assumptions* on the kernel *kerh* instead of *assigning* it to convey a sense of the *yet* unknown (or, *other*) without which the knowledge we have acquired becomes *unsustainable* before the powers that be.

How the kernel *kerh* is set—indeed the *awareness* of this task's existence and value—is of determining importance to meet dismissive attitudes to aspects of knowledge that appear locked to the deep end of genius (and therefore deemed unalienable): that is, the dismissal of experience because it is unfathomable. If instead of being assumed, the kernel *kerh* is assigned the unknown corpus of the original G, then we will screen this corpus from H: that is, it will be made to appear on the <u>scene</u> of what the image does *not* convey (instead of substituting the kernel for the image).

If the image *imh*, in semiotic terms, is iconic, then the kernel *kerh* is indexical. In which case they *cannot* be conflated. But this is on condition that homomorphism applies to investigations beyond mathematics. If the semiotic framework indeed invites such a development, there is still a problem that must be addressed at the level of doxa/assumption: **a**) that the art-field is excused from mathematics; **b**) that the humanities are exempt from mathematics; **c**) mathematics is not obligated to art and natural language. Can progressing in homomorphism contribute to right this wrong?

Clearly, the above assumptions are not water-tight, since coding is becoming increasingly common across the above fields. Yet, the same fields remain pervasively unassigned to one another. But rather than ascribing this to human stubbornness, and inertia, there might be an underlying model which is common to all: namely, that whatever is our professional field—in research academia—we operate based on the assumption that we are on board of a runaway train. Typing knowledge becomes conflated with the token, as we move on to the next publication. The train accelerates.



Fig. 3—Homomorphism in the study of crystals (crystallography): a branch of science devoted to the study of molecular and crystalline structure and properties, with applications in mineralogy, chemistry, physics, mathematics, biology, metallurgy and materials science. What we have attempted to show here is the fruitfulness of using H to study the heterostructural aspects of G.

We pose as powerless before these developments. But we are not powerless. But the acceleration contributes to make potentially important work drown in what <u>Erdward Tufte</u> called PowerPointPhluff (2003 p. 4): "Slideware helps speakers to outline their talks, to retrieve and show diverse visual materials, and to communicate slides in talks, printed reports, and internet. And also to replace serious analysis with chartjunk, over-produced layouts, cheerleader logotypes and branding, and corny clip art. That is, PowerPointPhluff." Likely, if reflected in the terms of homomorphism as discussed here, the problem is that the kernel *kerh* is assumed rather than assigned.

The ideas discussed here have been developed from the comparison of *deep learning* in photogravure and AI. And also <u>bimodal search</u> as an editorial design.